① $y = \pm\sqrt{\frac{8-x^4}{2}}$ Find $\frac{dy}{dx}$

② The volume of a cone is a function of both the radius and the height, $V(r,h) = \frac{\pi r^2 h}{3}$.

ⓐ Find $\frac{\partial V}{\partial r}$ if the height is kept constant.

ⓑ Find $\frac{\partial V}{\partial h}$ if the radius is kept constant.

③ Give the formula for the chain rule.

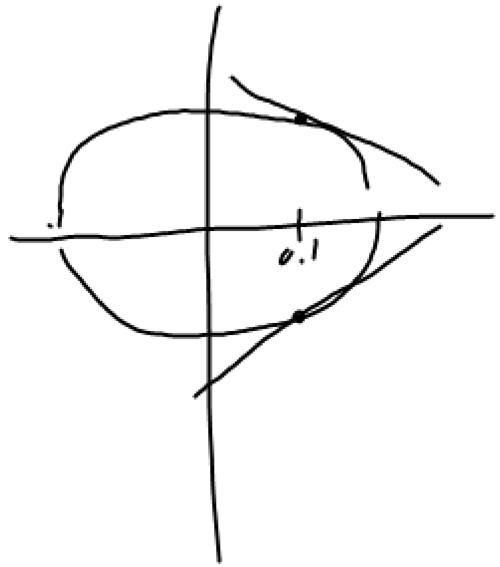
④ What does $\frac{dy}{dx}$ mean? Be thorough.

1. $\pm\sqrt{\dfrac{8-x^4}{2}} = \dfrac{\sqrt{8-x^4}}{\sqrt{2}} = (8-x^4)^{1/2}(2)^{-1/2}$

$y' = \left(2^{-1/2}\right) \tfrac{1}{2}(8-x^4)^{-1/2}\left(-4x^3\right)$

$y' = -2x^3(8-x^4)^{-1/2}\left(2^{-1/2}\right)$

$y' = \dfrac{\pm 2x^3}{\sqrt{8-x^4}\sqrt{2}}$



Keep h constant

$$\frac{\pi r^2 h}{3} \rightarrow \frac{\pi r^2 h}{3}$$

$$\frac{3 \cdot 2\pi h r}{9}$$

$$\frac{6\pi h r}{9} = \frac{2}{3}\pi h r$$

$$\frac{\pi r^2 h'}{3} \xrightarrow{\text{deriv}} \frac{\pi r^2}{0}$$

(Not a fraction, just Scott notation)

$$\frac{3\pi r^2}{9} = \frac{1}{3}\pi r^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$u^n \rightarrow n u^{n-1} \cdot \frac{du}{dx}$$

$\frac{dy}{dx} \Rightarrow$ derivative of y with respect to x, Slope of tangent $= \frac{\Delta y}{\Delta x}$ (change → Δ)

instantaneous rate of change, y'

$$\frac{dx}{dx} = 1$$

explicit equation

y on one side

$y = \pm\sqrt{1-x^2}$

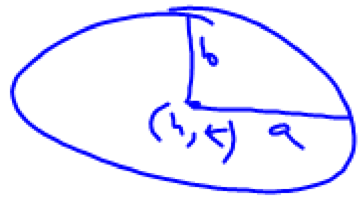
$y = mx + b$

implicit equation

x + y together

$x^2 + y^2 = 1$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



$y^5 - y = x$

$y^2 = x$

$2y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y}$

$\frac{dy}{dx} = nu^{n-1} \cdot \frac{du}{dx}$

y is some function of x

kind of like u was a function of x in chain rule

$x^4 + 2y^2 = 8$

$4x^3 + 4y\frac{dy}{dx} = 0$

$\frac{4y\frac{dy}{dx}}{4y} = \frac{-4x^3}{4y} \Rightarrow \frac{dy}{dx} = \frac{-x^3}{y}$

$$2y = x^2 + \sin y$$

$$2\frac{dy}{dx} = 2x + \cos y \frac{dy}{dx}$$

$$2\frac{dy}{dx} - \cos y \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}(2 - \cos y) = 2x \quad \rightarrow \quad \frac{dy}{dx} = \frac{2x}{2 - \cos y}$$

$$x^2 - xy + y^2 = 7$$

$$2x - \left(x \cdot 1 \frac{dy}{dx} + y \cdot 1\right) + 2y\frac{dy}{dx} = 0$$

$$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$

$$-x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x + y$$

$$\frac{dy}{dx}(-x + 2y) = -2x + y$$

slope function

$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

$$\left.\frac{dy}{dx}\right|_{(-1,2)} = \frac{2 - 2(-1)}{2(2) - (-1)} = \frac{4}{5}$$

Sect. 3.7

#1, 4, 5, 6, 7, 10, 11, 13, 16, 18, 19, 22, 23, 25